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Research Article

On the Dynamics and Stability of the Crime and Punishment Game

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We study the dynamics and stability of the economics of crime and punishment game from an evolutionary perspective. Specifically, we model the interaction between agents and controllers as an asymmetric game exploring the dynamics of the classic static model using a replicator dynamics equation, given exogenous levels of monitoring and criminal sanctions. The dynamics show five possible equilibria, from which three are stable. Our results show that a culture of honest agents is never stable; however when the penalty is high enough, the system will neutrally tend to an equilibrium of honest agents and a monitoring firm. By contrast, when the probability of detecting wrongdoing is small, the system, in some cases, will remain in a transient state, in which it is impossible to predict the proportion of honest agents.

1. Introduction

Corruption is a form of dishonesty undertaken by an individual or organization to acquire illicit benefit or abuse power for personal gain, deviating from established norms of behavior with or without legal or ethical connotations (see for instance [1, 2]).

It has always been an important concern for managerial teams in all kinds of organizations, private and public, being one of the main barriers to economic growth in many countries [3]. The World Economic Forum 1 estimated by 2018 that annual costs of corruption are about 5% of global GDP.

The classical theoretical studies on corruption, based on Becker's seminal work on crime and punishment [4, 5], hold that an offender decides whether to commit a crime and how much crime to commit by comparing the benefits and costs of crime with those of alternative activities.

In Becker's setting, an agent decides whether to commit a crime by comparing the expected utility of the offenses with those of alternative activities. In the context of a firm, an agent (or employee) will be honest whenever its incentive constraint is satisfied:

$$U(w) > (1-\theta)U(w+\beta) + \theta U(w_0 - f), \tag{1}$$

where w represents salary, β represents the offenses, θ is the probability of being caught, w_0 is the alternative salary if fired, f represents the fine an agent must pay if caught in criminal acts, and $U(\cdot)$ is the expected utility.

The implication of equation (1) is that an agent's incentive to be honest depends on its "efficiency wage" $(w - w_0)$ and the probability of detecting offenses, θ. Nevertheless, the model does not pay attention to the optimal value of the agent's wage or the optimal level of monitoring. Indeed, in the model, if $\theta = 1$, equation (1) is always satisfied for positive efficiency wages; by contrast, if $\theta = 0$, equation (1) is never satisfied. Paper [6] pointed out that for any reasonable distribution of wages, implying a lower frequency of extreme values, the more extreme the values taken by the monitoring probability θ, the lower the effect of wages on offenses.

Following Becker's approach, we assume "consensus" on the variables under study and we just try to find some guidelines for the treatment of offenses. Our approach is to analyze the dynamics of the classic game of criminal behavior using first an asymmetric 2-player game to model an organization in which agents and controllers coexist. Particularly, we focus our study on the long-term interaction between agents and controllers and their response to different incentives, by using the replicator dynamics equation to model the evolutionary dynamics of the problem of potential criminals (for an explanation of the replicator equation, see for instance [7, 8]). The solution of the system is given by the set of evolutionary steady states of the honest population and the probability of monitoring. Besides analyzing the traditional Beckerian model, we also model the dynamics of optimal enforcement policies.

The remainder of this paper is organized as follows. Section 2 presents a literature review. In Section 3 we present our proposed model in its static and dynamic versions and its evolutionary dynamics and contextualize our contribution. Section 4 analyzes the conditions under which the equilibria are stable and under what conditions the system is unstable. Section 5 provides some numerical simulations in which we use elements of chaos theory to analyze our replicator dynamics system in order to evaluate how sensitive to initial conditions this system is. Finally, Section 6 puts forward some concluding remarks and future research.

2. Literature Review

Our work relates to some different literature. Firstly, there is a large body of literature focusing on the design of rules, punishments, and control structures to discourage undesirable behavior in organizations. This literature, beginning with the seminal work of [4, 5] and its many successors (see, for example, [9–12]), has a main aim to study to a certain extent how the probability of being caught, the magnitude of the punishment, the proceeds of criminal activity, and the return to work (alternative to crime) would affect the level of crime. See [13] for a complete review.

Several extensions have been made to Becker's original work; however, the economics of crime maintains its original spirit. One interesting variation is made by [14] and used latter by [15]. This approach models explicitly the probability of being caught and fined using a smooth function p(m, x), where m represents the outlays devoted by society to monitoring and fining individuals and where x represents the individual's outlays on avoidance. With this formulation, it is possible to determine the optimal enforcement with screening for Becker's problem.

Another branch of this literature, also related to our study, shows how corruption and other activities become profitable when others also engage in such behavior, potentially leading to multiple equilibria. See for instance [16–18].

Our work is also related to those modeling the evolution of crime. For instance, [19] studies the consequences of asymmetric and symmetric penalties by developing deterministic and stochastic evolutionary game-theoretic models of bribery, finding that a reduction in incidents of bribery when using asymmetric penalties depends on how the players update their strategies over time. Similarly, [20] models the evolution of a system of corruption where players play a series of supergames with randomly chosen opponents. They find the conditions under which a population of adaptive and nonadaptive players lead to a corruption equilibrium which is stable. However, when corruption causes small but cumulative social costs, the corruption equilibrium becomes unstable and a new equilibrium of conditional honest players can be reached. Finally, [21] models the dynamics of corruption using the stability theory of differential equations. The model exhibits forward bifurcation. They reformulate the model as an optimal control problem, with the use of two time-dependent controls to assess the impact of corruption on human population, finding that an integrated control strategy must be followed to successfully fight corruption.

A more recent branch of research that is more closely related to our work investigates the dynamics of crime. Paper [22] studies the relationship between corruption in public procurement and economic growth. In particular, the authors implement a Solow growth model in discrete time, assuming that the public good is an input and that the state fixes a monitoring level on corruption. They find multiple equilibria, perform a stability analysis, and prove the existence of a compact global attractor. The main finding is that no long-run equilibria with zero corruption exist and, furthermore, that periodic or aperiodic fluctuations in economic growth are likely to emerge. Paper [23] put forward a model of crime transmission to study the spread of crime, imprisonment, and recidivism. One of the main findings of this work is that the long-run effect of increasing the length of prison terms has no effect on changing the fraction of criminally active people in the population, which contrasts findings from [24] who propose a model of crime transmission inherited with memory property. They show the progression of the flow of population by classifying into three systems based on involvement in crime and imprisonment by considering the criminal history of an individual. They conclude that both high and low crime-equilibria exist and that a crime-free equilibrium can also be found. They show that a minute growth in the imprisonment rate tends to lower the spread of crime. Paper [25] also proposes a fractional-order crime transmission model with four types of citizens. Paper [26] proposes a differential equation model to analyze whether more legal guns mean less crime committed by illegal guns. The results of this work show that strong gun control does not ensure a crime-free society. On the contrary, weak gun control can lead to a crime-free society; however, this policy requires the maximum number of legal guns in the hands of civil society. Paper [27] proposes a bimatrix game between property owners and criminals (or potential criminals) to analyze the private effort made by property owners, given a certain level of enforcement and magnitude of penalties. They conclude that high penalties and greater police enforcement not always deter crime in the long-term. They also show a cyclical behavior of crime over time. Paper [28] analyzes the dynamics of crime considering that punishments are costly, and therefore, using sequential games, they examine optimal enforcement of law strategies that minimize the effective punishments required to deter crime. In [23] instead of analyzing the dynamics of crime at the macro level, they do so at the micro level and study "the criminal career" and its dynamics with respect to punishment. They propose a model in which a criminal can be in one of five stages: (1) no criminal behavior, (2) criminal behavior and never incarcerated, (3) incarcerated, (4) repeat offenders, and (5) released. They find conditions leading to crime-free equilibrium when citizens can turn to crime only through contagion. However, when citizens can also turn to crime by their own, a crime-free society is not possible.

On the empirical side, [29] uses data of one specific Swedish street gang composed by three different datasets, to develop different social networks analysis and compare them by computing distance, centrality, and clustering measures. They show that different data sources about the same object of study have a fundamental impact on the results. Paper [30] presents an empirical approach to model corruption using complex networks. They describe a major corruption scandal in Mexico involving a network of hundreds of shell companies. Their analysis offers insight into the systemic nature of corruption and the shortcomings of reductionist analyses.

Our study is close to the theoretical models outlined in the review. Using a two-population dynamic replicator, we model the classical variables mentioned in the economic literature to control corruption in an organization, such as wage, monitoring, and penalties to study to what extent these variables discourage crime and under what conditions the equilibria found are stable.

3. The Model

For model specification, multiple populations can be used (see, for example, [31]). To simplify notation, we model a two-population system, where we will model the interaction between employees and controllers within an organization. Let us define G as a static asymmetric 2-player game defined by $G = (S_a, S_c, u_{ac}, v_{ac})$, where S_i is a finite set of actions for player i; u_{ij}: $S_a \times S_c \longrightarrow \mathfrak{R}$ and v_{ij}: $S_a \times S_c \longrightarrow \mathfrak{R}$ are their payoff functions. Player 1 is an employee and player 2 is a controller within the firm, who is in charge of deterring wrongdoing. Each player in this game has two available strategies. For the employee, the strategies are to behave honestly, h, or to behave corruptly, c, represented by $S_a = \{h, c\}$, and for a controller, representing the firm, the strategies are to monitor or not to monitor, represented by $S_c = \{m, nm\}$. Correspondingly, Δ_a and Δ_c stand for their respective spaces of mixed strategies. The payoff matrices U, $V \in \mathfrak{R}^{2x2}$ are defined for the employee and the controller, respectively.

The notation is presented in Table 1, and Table 2 summarizes the payoffs of the game.

Proposition 1. *Considering the asymmetric two-player game G described above, we obtain the following results:*

(1) There is a unique Nash equilibrium given by the criminal behavior of the employee and a monitoring firm $\{c, m\}$ whenever $v_{cnm} < v_{cm}$ and $u_h < u_{cm}$

(2) There is a unique Nash equilibrium given by the criminal behavior of the employee and a non-monitoring firm $\{c, nm\}$ whenever $v_{cnm} > v_{cm}$

(3) The game also has a single mixed-strategy equilibrium whenever $v_{cnm} < v_{cm}$ and $u_h > u_{cm}$, with a probability of an employee being honest of $p = 1 - m/\theta(\alpha\beta + f)$ and a probability of a monitoring firm of $s = \beta/\theta(w + \beta + f - w_0)$

Proposition 1.1 implies that $f \in ((m/\theta) - \alpha\beta, (\beta/\theta) - w - \beta + w_0)$; that is, if earnings per wrongdoing β are high enough and the probability $\theta \longrightarrow 0$, there will be no penalty high enough to stop the offenses. By contrast, Proposition 1.2 implies that $f < m/\theta - \alpha\beta$, which means that when the penalty is rather low, the firm will not monitor, and agents will behave corruptly. Proposition 1.3 implies that if $f > (\beta/\theta) - w - \beta + w_0$ whenever $(\beta/\theta) - w - \beta + w_0 > (m/\theta) - \alpha\beta$ or if $f > (m/\theta) - \alpha\beta$ whenever $(\beta/\theta) - w - \beta + w_0 < (m/\theta) - \alpha\beta$, the two players have no pure strategies which are optimal responses to each other. The payoff structure implies that an employee will commit a crime if not monitored and behave honestly if monitored. In contrast, the controller monitors if employees commit a crime and does not monitor if employees do not commit a crime. This implies that there is not a "dominant strategy." This in turn means that there is no best decision regardless of the decision of the opponent. However, we found a mixed strategy for the probability distribution of the employee behavior $(p, 1 - p)$ and the probability distribution of the controller decision to monitor or not $(s, 1 - s)$ of their set of pure strategies. The expected payoff of an employee and the controller are as follows:

$$\begin{aligned} u_{\text{mixed}} &= p u_h + (1 - p)\left[(1 - s)u_{cnm} + s u_{cm}\right] \\ v_{\text{mixed}} &= (1 - s)(1 - p)v_{cnm} + s\left[p v_{hm} + (1 - p)v_{cm}\right]. \end{aligned} \tag{2}$$

From equation (2), we can determine the proportion of honest agents and the probability that a controller monitors or not:

$$\begin{aligned} \frac{\partial u_{\text{mixed}}}{\partial p} &= \theta s\left(w + \beta + f - w_0\right) - \beta \\ \frac{\partial v_{\text{mixed}}}{\partial s} &= \theta(1 - p)(\alpha\beta + f) - m. \end{aligned} \tag{3}$$

From equation (3) we can conclude that

$$\begin{aligned} p^* &= 1 - \frac{m}{\theta(\alpha\beta + f)} \\ s^* &= \frac{\beta}{\theta(w + \beta + f - w_0)}. \end{aligned} \tag{4}$$

Nevertheless, the relationship between agents and controllers is a continuous and long-term relationship. This

Table 1: Notation.

w	Wages
w_0	Alternative wage when fired/opportunity cost
β	Earnings per wrongdoing
m	Monitoring costs
f	Penalty if detected in wrongdoing
α	Proportion of the recovered earnings per wrongdoing when a criminal is caught
θ	Probability of detecting wrongdoing

Table 2: Expected payoffs random matching game. m

	No monitoring	Monitoring
Honest	$u_h = w$, $v_{hnm} = 0$	$u_h = w$, $v_{hm} = -m$
Criminal	$u_{cnm} = w + \beta$, $v_{cnm} = -\beta$	$u_{cm} = (1-\theta)(w+\beta) + \theta(w_0 - f)$, $v_{cm} = \theta(f + \alpha\beta) - \beta - m$

m represents monitoring costs and α is a proportion of the recovered earnings per wrongdoing when a criminal is caught.

relationship will not necessarily conform to optimal proportions of honest/corrupt and controllers who choose to monitor or not just by playing the game. This adjustment is a complex and dynamic process. In reality, information is incomplete for both agents and controllers, and obtaining additional information is always costly. In terms of our model, we assume that while individuals are economic agents pursuing material benefits, in some cases individuals do not act rationally, at least from the perspective of the economy as a whole, that is, trying to achieve maximum social benefit. In this sense, we model the dynamics of agents and controllers actions using the replicator dynamics equation, which explicitly models the process by which the frequency of a strategy changes in the population, allowing us to study the evolutionary dynamics of the model. In the context of the replicator dynamics, individuals imitate the strategies of randomly sampled members of the population with a probability proportional to the difference in gains between the players, as long as the difference is positive. Assuming that individuals always imitate the best performing agents, otherwise the dynamics change. The replicator dynamics can be interpreted as a model of bounded-rationality, in which individuals learn about the game on a trial and error basis and where more efficient behavior, in evolutionary terms, tends to be imitated. This approach allows us to analyze the impact of different initial shares of agent and controller populations and the relevance of the parameters in explaining the stability (or instability) of corruption.

Consider the game G described above, where strategies S_a and S_c in the infinite sequence of action profiles are represented by (a, c). At each period $t \in [0, \infty)$ every member of each population is randomly matched with an individual from the other population to play a bilateral finite game with payoff matrices given by U and V, where $u_{ij} = \pi^a(s_i^a, s_j^c)$ and $v_{ij} = \pi^c(s_i^a, s_j^c)$ stand for the payoff obtained by the agent and the firm, respectively, if the former adopts strategy s_i^a and the latter strategy s_j^c.

Consequently, the dynamic adjustment process of our two-population replicator equations can be represented as follows:

$$\begin{aligned} \frac{\mathrm{d}p}{\mathrm{d}t} &= p\left[su_h + (1-s)u_h - u\right] \\ \frac{\mathrm{d}s}{\mathrm{d}t} &= s\left[pv_{hm} + (1-p)v_{cm} - v\right], \end{aligned} \tag{5}$$

where p represents the proportion of honest agents (and then $1-p$ represents the proportion of criminals), and s represents the proportion of controllers who do monitor (and therefore $1-s$ is the proportion of controllers who do not monitor). It should be noted that these ratios can also be viewed as the probabilities associated with honest/corrupt agents and monitor/do not monitor. Finally, u and v are the expected payoff to the two players when the agent uses mixed strategy $S_a \in \Delta_a$ and the controller uses mixed strategy $S_c \in \Delta_c$, with $u = s[pu_h + (1-p)u_{cm}] + (1-s)[pu_h + (1-p)u_{cnm}]$ and $v = p[sv_{hm} + (1-s)v_{hnm}] + (1-p)[sv_{cm} + (1-s)v_{cnm}]$.

By solving the system $(\mathrm{d}p/\mathrm{d}t) = 0$, $(\mathrm{d}s/\mathrm{d}t) = 0$, we get 5 equilibria solutions (p^*, s^*) in which the system converges (fixed points of the system). E_1, E_2, and E_3 represent the Nash equilibria of the static game (Proposition 1), where $E_1 = (0,0)$ represents environment where all agents are corrupt and the firm does not monitor, $E_2 = (0,1)$ in which all agents are corrupt and the firm always monitors, and $E_3 = (1 - (m/\theta(\alpha\beta + f)), \beta/\theta(w + \beta + f - w_0))$, where the proportion $1 - (m/\theta(\alpha\beta + f))$ of agents are honest and the firm monitors with probability $(\beta/\theta(w + \beta + f - w_0))$. Additionally, we get $E_4 = (1,0)$, representing an organization where all agents are honest and the firm does not monitor and $E_5 = (1,1)$, where all agents are honest, but the firm still monitors at all times.

4. Analysis of Equilibria

The Jacobian matrix for the system presented in equation (5) is as follows:

$$J = \begin{bmatrix} \begin{array}{c} -p\left(s\theta\left(w+\beta+f-w_0\right)-\beta\right)+ \\ (1-p)\left(s\theta\left(w+\beta+f-w_0\right)-\beta\right) \end{array} & \theta p(1-p)\left(w+\beta+f-w_0\right) \\ -\theta s(1-s)(\alpha\beta+f) & \begin{array}{c} s(-\theta(1-p)(\alpha\beta+f)+m)+ \\ (1-s)(\theta(1-p)(\alpha\beta+f)-m) \end{array} \end{bmatrix}. \tag{6}$$

The approach we follow to study the stability of each point is by means of the analysis of eigenvalues from the Jacobian matrix, which means imposing the condition $\lambda_i < 0 \forall i$.

By solving the characteristic polynomial of the Jacobian (equation (6)) with fixed point E_1, the eigenvalues are

$$\lambda_1 = -\beta,\ \lambda_2 = \theta(\alpha\beta + f) - m. \tag{7}$$

Since $\beta > 0$, E_1 to be stable must satisfy that $\lambda_2 < 0 \Leftrightarrow f < (m/\theta) - \alpha\beta$, which is consistent with the Nash equilibrium condition.

When solving equation (6) with fixed point E_2, the eigenvalues are

$$\lambda_1 = \theta\left(\beta + f + w - w_0\right) - \beta,\ \lambda_2 = m - \theta(\alpha\beta - f). \tag{8}$$

Imposing the condition $\lambda_1 < 0$ and $\lambda_2 < 0$, we get that E_2 is a stable fixed point if $f \in ((m/\theta) - \alpha\beta, (\beta/\theta) - w - \beta + w_0)$ and $(m/\theta) - \alpha\beta < (\beta/\theta) - w - \beta + w_0$, which is also consistent with Proposition 1.

For fixed point E_3, the eigenvalues of the Jacobian represented in (6) are given by

$$\lambda_{1,2} = \pm \frac{\sqrt{-m\beta\left(w+\beta+f-w_0\right)(\alpha\beta+f)(\theta(w+\beta+f-w0)-\beta)(\theta(\alpha\beta+f)-m)}}{\theta(\alpha\beta+f)\left(w+\beta+f-w_0\right)}. \tag{9}$$

In this case, when $f > (\beta/\theta) - w - \beta - w_0$ and $-m\beta(w + \beta + f - w_0)(\alpha\beta + f)(\theta(w + \beta + f - w0) - \beta)(\theta(\alpha\beta + f) - m) > 0$, the equilibrium will be a saddle point. On the other side, if $f > (\beta/\theta) - w - \beta - w_0$ and $-m\beta(w + \beta + f - w_0)(\alpha\beta + f)(\theta(w + \beta + f - w0) - \beta)(\theta(\alpha\beta + f) - m) < 0$, the eigenvalues are purely imaginary numbers, which means that the system will oscillate around the equilibrium with constant amplitude, getting a neutrally stable equilibrium.

The eigenvalues when evaluating the Jacobian at the point E_4 are

$$\lambda_1 = -m,\ \lambda_2 = \beta. \tag{10}$$

Since $m, \beta > 0 E_4$ is a saddle point.

Finally, the eigenvalues associated with E_5 are

$$\lambda_1 = m,\ \lambda_2 = -\theta\left(w + \beta + f - w_0\right) + \beta. \tag{11}$$

If $\lambda_2 > 0$, E_5 represents an unstable node and when $\lambda_2 < 0$, E_5 represents a saddle point.

It is not surprising that an equilibrium of honest agents (E_4 and E_5) is not stable, since in an asymmetric game only pure-strategy Nash equilibria can be asymptotically stable (see [32]). The Nash equilibrium represents a situation in which an individual has no incentives to change its strategies, assuming the other players remain constant in their actions. Thus, the results found point to the fact that there is not a clear cut solution to beat crime, which cannot be a surprise, since crime has been a conduct difficult to break since the beginning of time.

5. Numerical Simulation

In this section we present some numerical examples of the model to illustrate some of our theoretical results. In particular, we are interested in simulating the parameters associated with earnings per wrongdoing β, the penalty f, the proportion of the recovered earnings per wrongdoing α, the probability θ, and the cost of monitoring m. For a better understanding of our model of crime and punishment and its chaotic behavior, we analyze the long-term game of system depicted in 5 by using initial value sensitivity and bifurcation diagrams.

Note that, in all the experiments, we use the wage as numeraire. In other words, we set the value of the wage to 1 and for each experiment we made the value of a parameter vary. For instance, when we vary the parameter β between 0.1 and 4, this implies that it varies in terms relative to the wage between 10% and 400%.

5.1. Initial Value Sensitivity. Figure 1 shows two examples of basin of attraction of the equilibrium points, the left graph when $f = 1.5$ and the right graph when $f = 3$. There are coexisting attractors, where the red, blue, green, purple, and pink regions are the feasible basin of attraction for E_1, E_2, E_3, E_4, and E_5, respectively. The axes represent the initial value taken by system 5.

5.2. 1D Diagram Bifurcation. In Figure 2, we simulate values of $\beta \in [0, 4]$. E_1 is never stable, since it must satisfy that

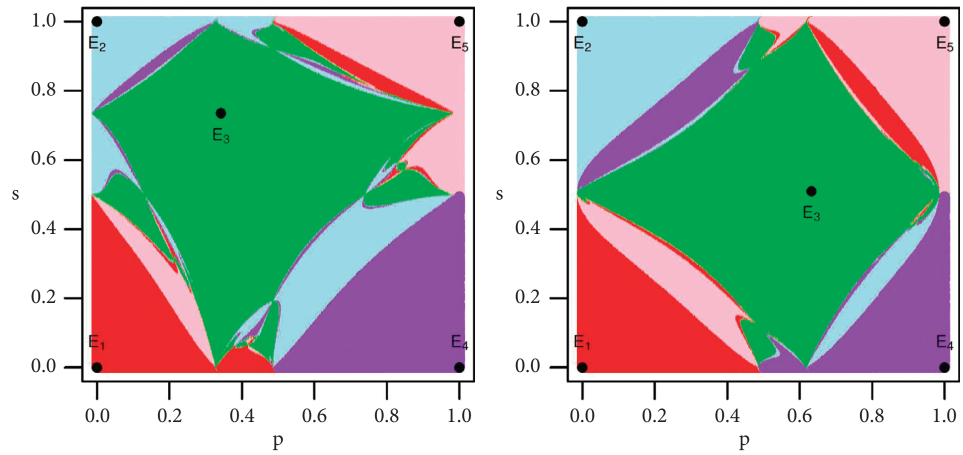

Figure 1: The basin of attraction when $w = 1$, $w_0 = 0.1$, $\beta = 1$, $m = 0.5$, $\alpha = 0.4$, $\theta = 0.4$. Left: $f = 1.5$. Right: $f = 3$.

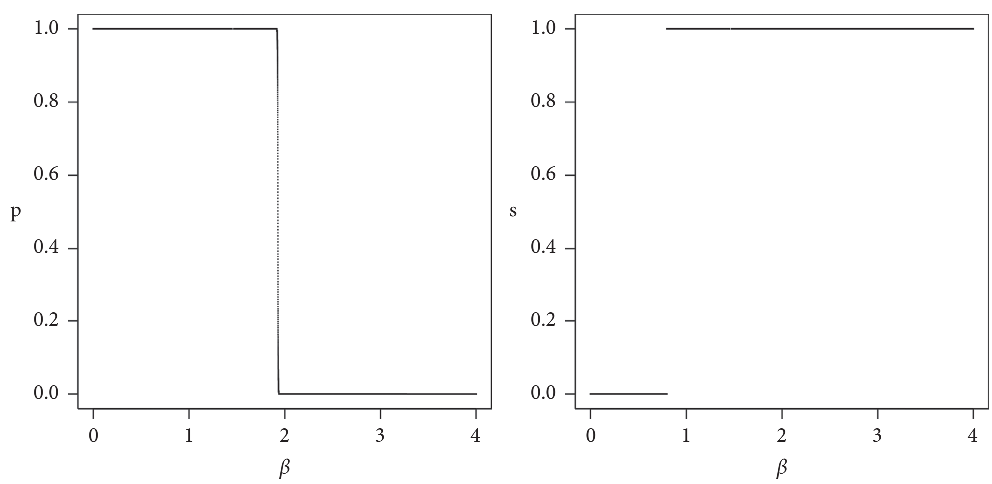

Figure 2: The bifurcation cascade for honest agents and the probability of monitoring as a function of earnings per wrongdoing. Parameters: $w_0 = 0.1$, $m = 0.1$, $f = 2$, $\alpha = 0.4$, $\theta = 0.4$.

$\beta < -4.375$. The fixed point E_2 is stable when $\beta > 1.9\overline{3}$. When $\beta \in (0, 1.9\overline{3})$, the eigenvalues for fixed point E_3 are always pure imaginary numbers, and then an oscillation around the equilibrium point $(1 - (0.25/0.4\beta + 2), (2.5\beta/2.9 + \beta))$. In Figure 3, we simulate the proportion of honest agents and the probability the firm will monitor for $\beta = 0.3$, $\beta = 1.1$, and $\beta = 3$, showing that for small values of β the firm does not monitor and for high values, although the firm always monitors, the agents are corrupt.

We simulate values for $f \in [0, 4]$. Figures 4 and 5 show graphically what would happen with the organization, setting different values for the fine f. E_1 is never stable, since it must satisfy that $f < -0.35$. The fixed point E_2 is stable when $f < 1.35$. The eigenvalues for fixed point E_3 are always pure imaginary numbers. In Figure 5 we simulate the proportion of honest agents and the probability the firm will monitor for $f = 0.8$, $f = 2$, and $f = 4$, showing that for small values of f the firm monitors, but agents are corrupt. For a value equal to twice the wage, there is instability, in which case, high fines do not fulfill the purpose of deterring corruption. Only when fines are extremely high (for example, f four times the wages), the agents become honest and the firm will not monitor.

We simulate values for $\alpha \in [0, 1]$. Figures 6 and 7 show that, for our simulations, α has no major impact on the behavior of agents or the firm. The proportion of recovered earnings per wrongdoing is not a determinant of chaos in our model. The model converges to E_2 for all values of α under the parameters of our simulation. In Figure 7, we simulate the proportion of honest agents and the probability the firm will monitor for $\alpha = 0.05$, $\alpha = 0.2$, and $\alpha = 0.8$, showing the same convergence to E_2 for all values of the proportion of recovered earnings per wrongdoing.

We simulate values for $\theta \in [0, 1]$. Figures 8 and 9 show the evolution of an organization by varying the auditing probability of detecting wrongdoing θ. For values of $\theta < 0.\overline{037}$, the system converges to the equilibrium E_1. When $\theta \in (0.\overline{037}, 0.\overline{5})$, the system converges to E_2, and for values of

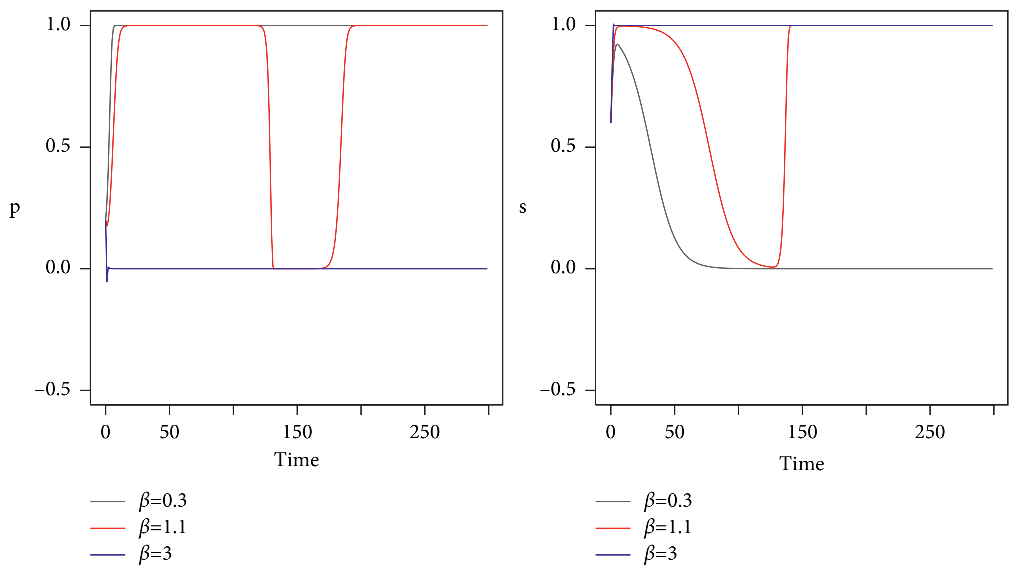

FIGURE 3: Time series of system for $\beta = 0.3$, $\beta = 1.1$, and $\beta = 3$. Parameters: $w_0 = 0.1$, $m = 0.1$, $f = 2$, $\alpha = 0.4$, $\theta = 0.4$.

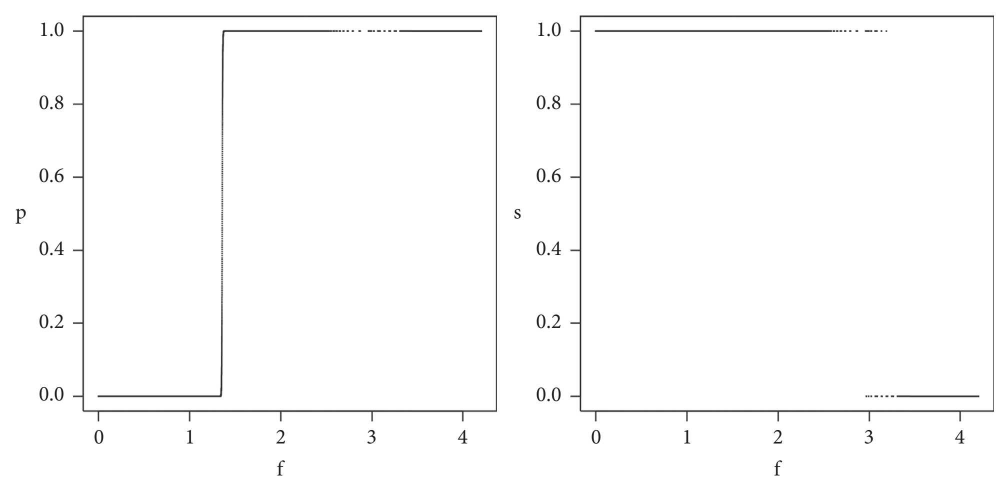

FIGURE 4: The bifurcation cascade for honest agents and the probability of monitoring as a function of the penalty. Parameters: $w_0 = 0.1$, $m = 0.1$, $\beta = 1.5$, $\alpha = 0.4$, $\theta = 0.4$.

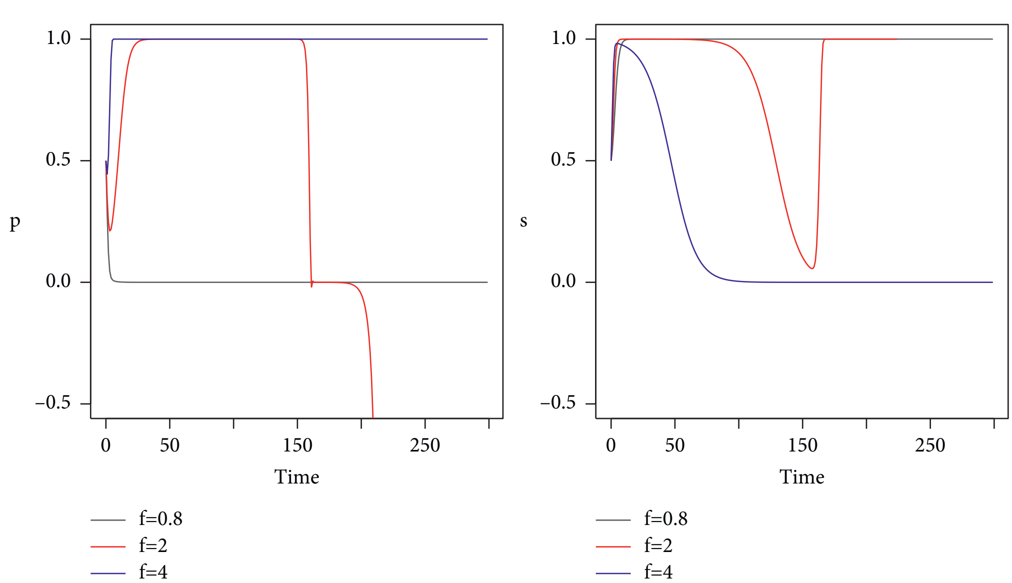

FIGURE 5: Time series of system for $f = 0.8$, $f = 2$, and $f = 4$. Parameters: $w_0 = 0.1$, $m = 0.1$, $\beta = 1.5$, $\alpha = 0.4$, $\theta = 0.4$.

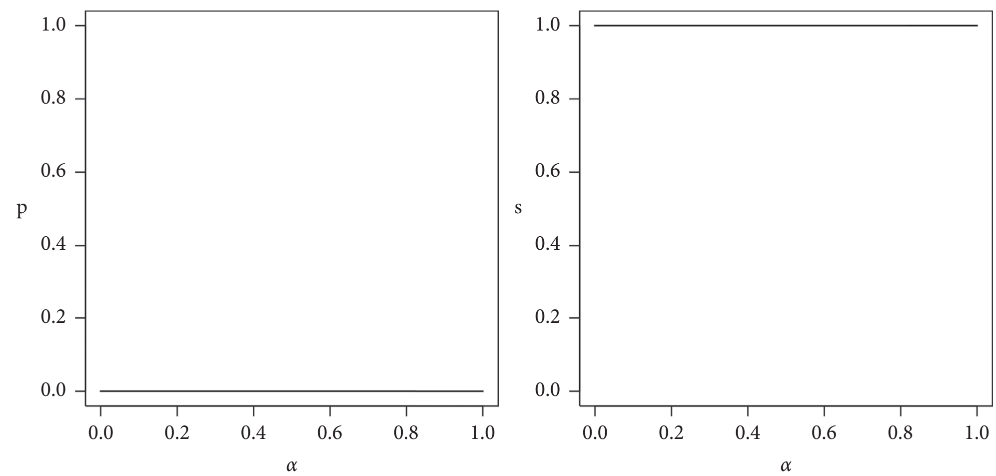

FIGURE 6: The bifurcation cascade for honest agents and the probability of monitoring as a function of the proportion of recovered earnings per wrongdoing. Parameters: $w_0 = 0.1$, $m = 0.1$, $\beta = 3$, $f = 1.5$, $\theta = 0.4$.

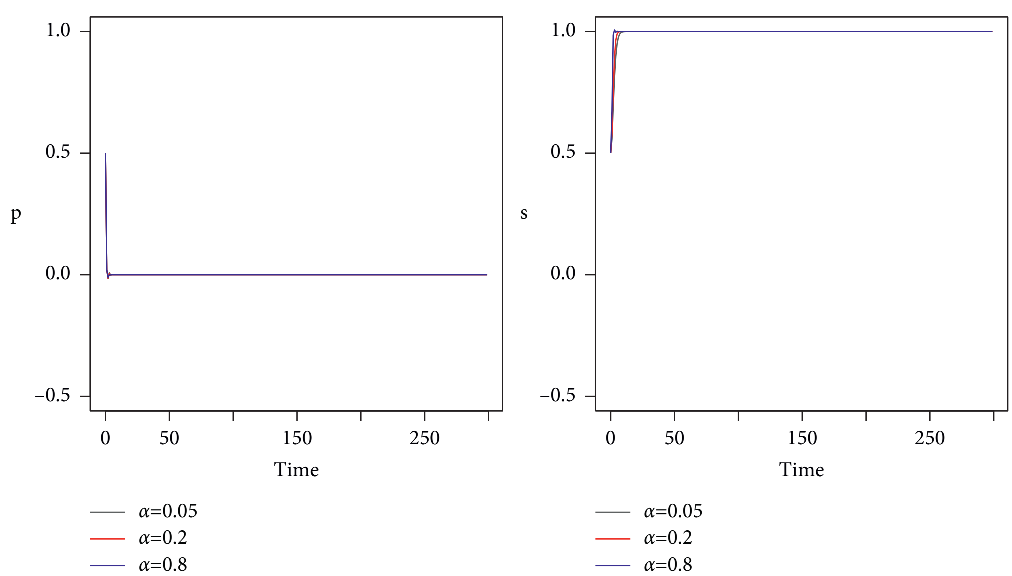

FIGURE 7: Time series of system for $\theta = 0.05$, $\theta = 0.15$, and $\theta = 0.4$. Parameters: $w_0 = 0.1$, $m = 0.1$, $\beta = 3$, $f = 1.5$, $\theta = 0.4$.

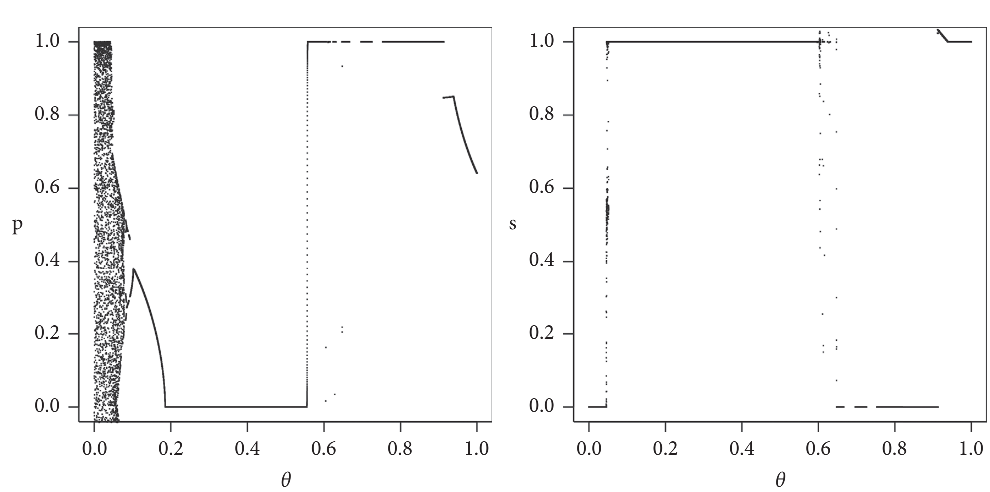

FIGURE 8: The bifurcation cascade for honest agents and the probability of monitoring as a function of the probability of catching wrongdoing. Parameters: $w_0 = 0.1$, $m = 0.1$, $\beta = 3$, $f = 1.5$, $\alpha = 0.4$.

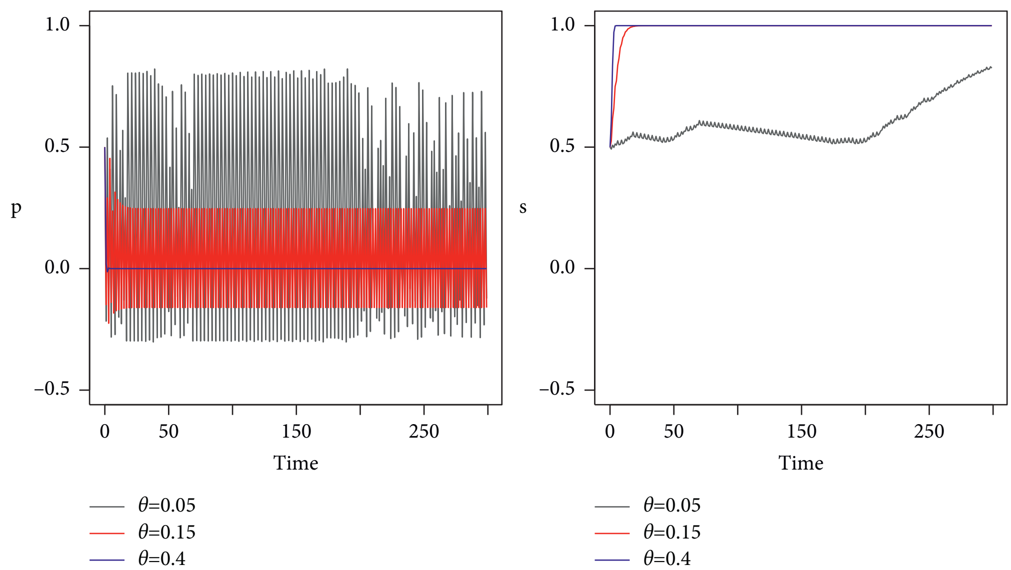

Figure 9: Time series of system for $\theta = 0.05$, $\theta = 0.15$, and $\theta = 0.4$. Parameters: $w_0 = 0.1$, $m = 0.1$, $\beta = 3$, $f = 1.5$, $\alpha = 0.4$.

$\theta > 0.\overline{5}$, the eigenvalues associated with E_3 are pure imaginary number, which means that the system oscillates around the point $(1 - (0.\overline{037}/\theta), (0.\overline{5}/\theta))$.

However, Figure 8 shows chaos for the proportion of honest agents when $\theta < 0.18$, which is not predicted by the analysis of the eigenvalues. What is happening in this zone of small values of θ is that the rate at which the system converges to equilibrium E_1 (when $\theta < 0.\overline{037}$) and to E_2 (when $\theta \in (0.\overline{037}, 0.18)$) tends to infinity, and therefore, the system will reach equilibrium in infinite time. Figure 10 shows in panel (a) the phase diagram when $\theta = 0.01$, panel (b) when $\theta = 0.09$, panel (c) when $\theta = 0.4$, and finally panel (d) when $\theta = 0.7$, where we show the differences in convergence of these four values for the probability of detecting wrongdoing θ.

We simulate values for the cost of monitoring $m \in [0, 4]$. Figure 11 shows the 1D diagram bifurcation and Figure 12 shows for three values of monitoring costs their evolution through time. For our simulation, the system converges to E_2 whenever $f < 1.08$. However, the condition for E_1 to be stable and E_3 to be a neutrally stable equilibrium is the same ($m > 1.08$), which means that in some zones the equilibrium will become E_1 and in others, it will be E_3. As shown in Figure 11, for $m > 1.08$ the proportion of honest agents is unstable. The probability of monitoring is zero when $m \in [1, 2.65]$ and unstable for values of $m > 2.65$.

5.3. 2D Bifurcation Diagram. We first study the limit cycle of the system for β versus θ, with the rest of the parameters fixed, being $w_0 = 0.1$, $m = 0.1$, $f = 1.5$, $\alpha = 0.4$, and for β versus f and the parameters $w_0 = 0.1$, $m = 0.1$, $\alpha = 0.4$, $\theta = 0.4$. Figure 13 shows the results. Panel (a) shows our findings when varying the values of β and θ simultaneously, where three infinite period bifurcations are found. In the limit cycle IPB_{123} collides with the equilibria E_1, E_2, and E_3. In the limit cycle IPB_{235} collides with the equilibria E_2, E_3, and E_5. Finally, in the limit cycle IPB_{134} collides with the equilibria E_1, E_3, and E_4. Panel (b) shows the limit cycles when we vary the values of β and f. We find four limit cycles, where IPB_{123} is an infinite period bifurcation where the equilibria E_1, E_2, and E_3 collide. In the limit cycle IPB_{235} collides with the equilibria E_2, E_3, and E_5. In the limit cycle IPB_{134} collides with the equilibria E_1, E_3, and E_4. Finally, SC_3 is the stable limit cycle of a supercritical Hopf bifurcation.

By means of 2D bifurcation diagrams, we analyze the areas in which the system converges to the fixed points and which areas are chaotic. Red area is used to represent the equilibrium point of corrupt agents and a nonmonitoring firm (E_1), light blue area for corrupt agents and a monitoring firm (E_2), green area for a proportion p^* of honest agents and a firm that monitors with probability s^*, purple for honest agents and a nonmonitoring firm (E_4), pink for the equilibrium of honest agents and a monitoring firm, and gray area for indicating chaotic region of the system.

Figure 14 shows four two-dimensional bifurcation diagrams. The upper left panel indicates the relation between probability of detecting wrongdoing and earnings per wrongdoing. We can conclude that when θ is relative small, there are no incentives to monitor, and therefore agents will become corrupt. When θ increases, there is a path of petty crime (small β), where the firm does not monitor. However when β becomes attractive and θ is not high enough, the path is of corruption and a monitoring firm, but as θ becomes larger, even for high amounts of β, the agents are honest. The upper right panel shows a similar pattern when we study the

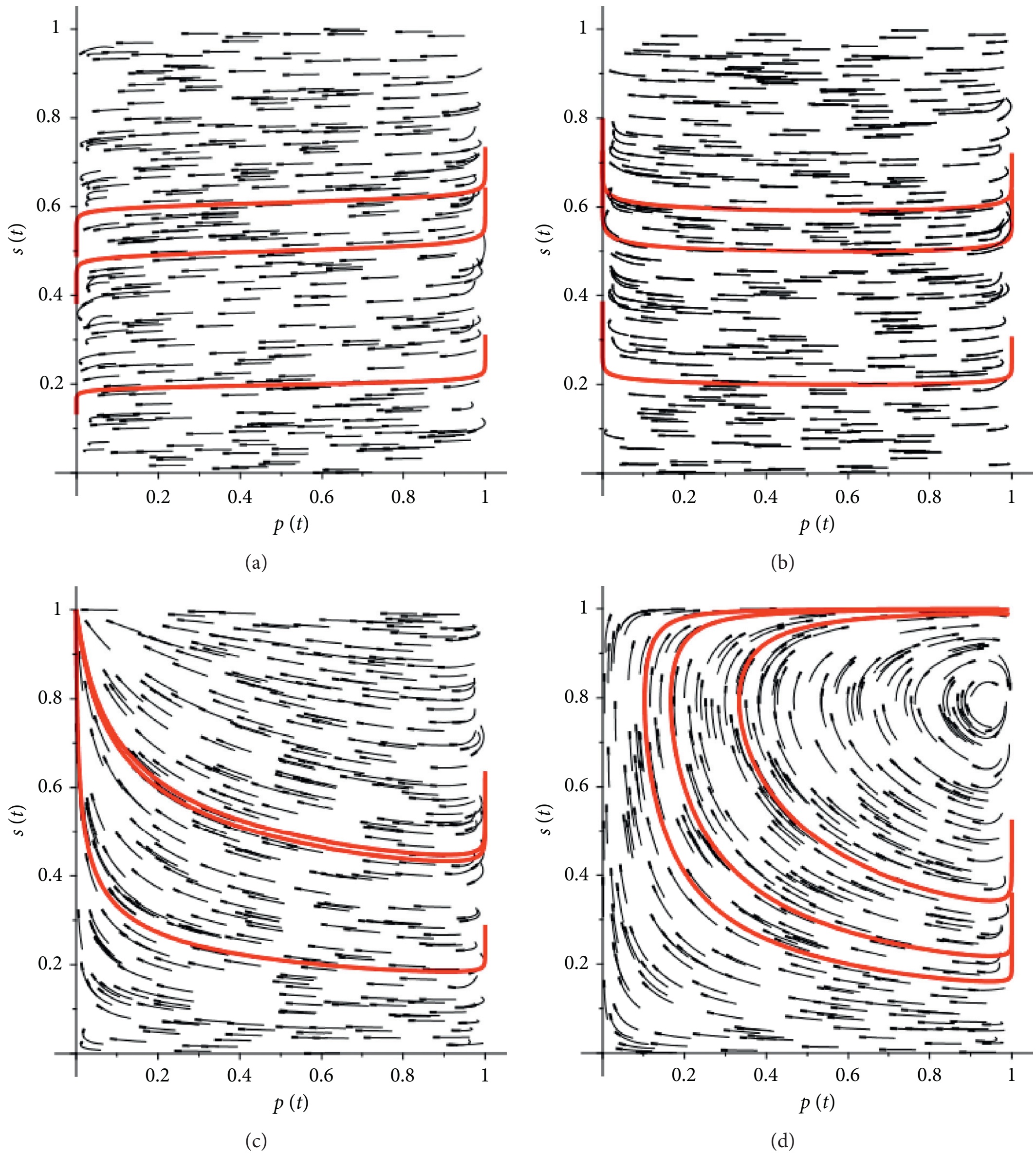

FIGURE 10: Phase diagram. Parameters: $w_0 = 0.1$, $m = 0.1$, $\beta = 3$, $f = 1.5$, $\alpha = 0.4$. (a) $\theta = 0.01$, convergence to E_1 when $t \longrightarrow \infty$, (b) $\theta = 0.09$, convergence to E_2 when $t \longrightarrow \infty$, (c) $\theta = 0.4$, convergence to E_2, (d) $\theta = 0.7$, convergence to E_3.

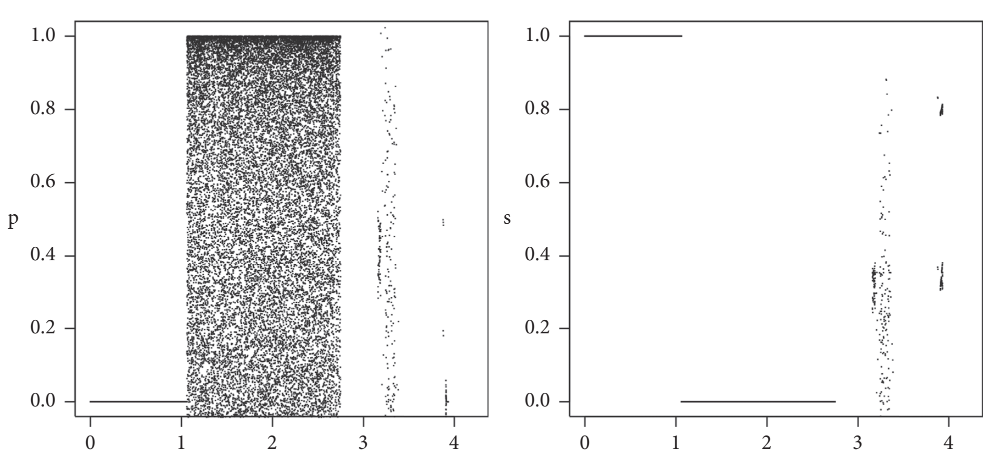

FIGURE 11: The bifurcation cascade for honest agents and the probability of monitoring as a function of monitoring costs. Parameters: $w_0 = 0.1$, $\beta = 3$, $f = 1.5$, $\alpha = 0.4$, $\theta = 0.4$.

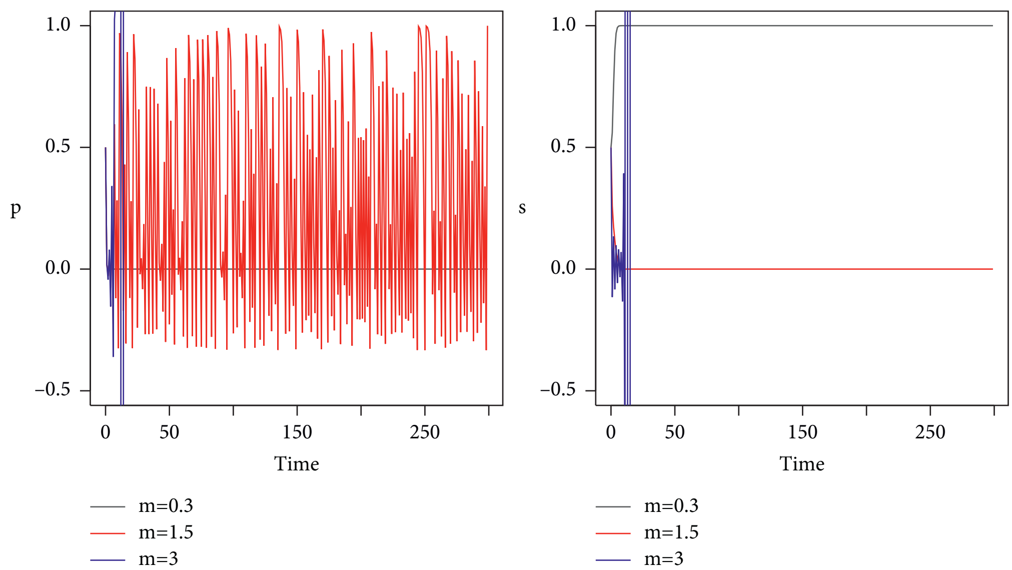

FIGURE 12: Time series of system for $m = 0.3$, $m = 1.5$, and $m = 3$. Parameters: $w_0 = 0.1$, $\beta = 3$, $f = 1.5$, $\alpha = 0.4$, $\theta = 0.4$.

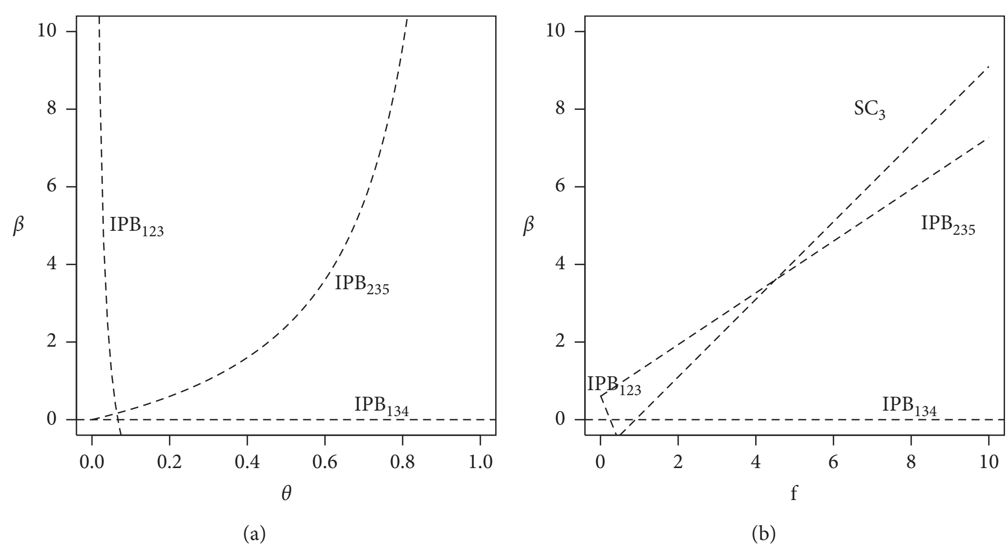

FIGURE 13: Limit cycle. Panel (a) $w_0 = 0.1$, $m = 0.1$, $f = 1.5$, $\alpha = 0.4$. Panel (b) $w_0 = 0.1$, $m = 0.1$, $\alpha = 0.4$, $\theta = 0.4$.

relation between β and the fine f. If β is small, f serves no purpose and regardless of the size of the fine, there will be petty crime. As β increases, the fine, in order to deter crime, must be very high; otherwise in the long-run there will only be corrupt agents and a firm that always monitors. However the challenge is that corrupt agents actually pay the penalties imposed. The lower left panel shows the relation between β and monitoring costs m. We conclude that if monitoring is cheap, then there is convergence towards an honest agents' condition. The lower right panel shows the relation between

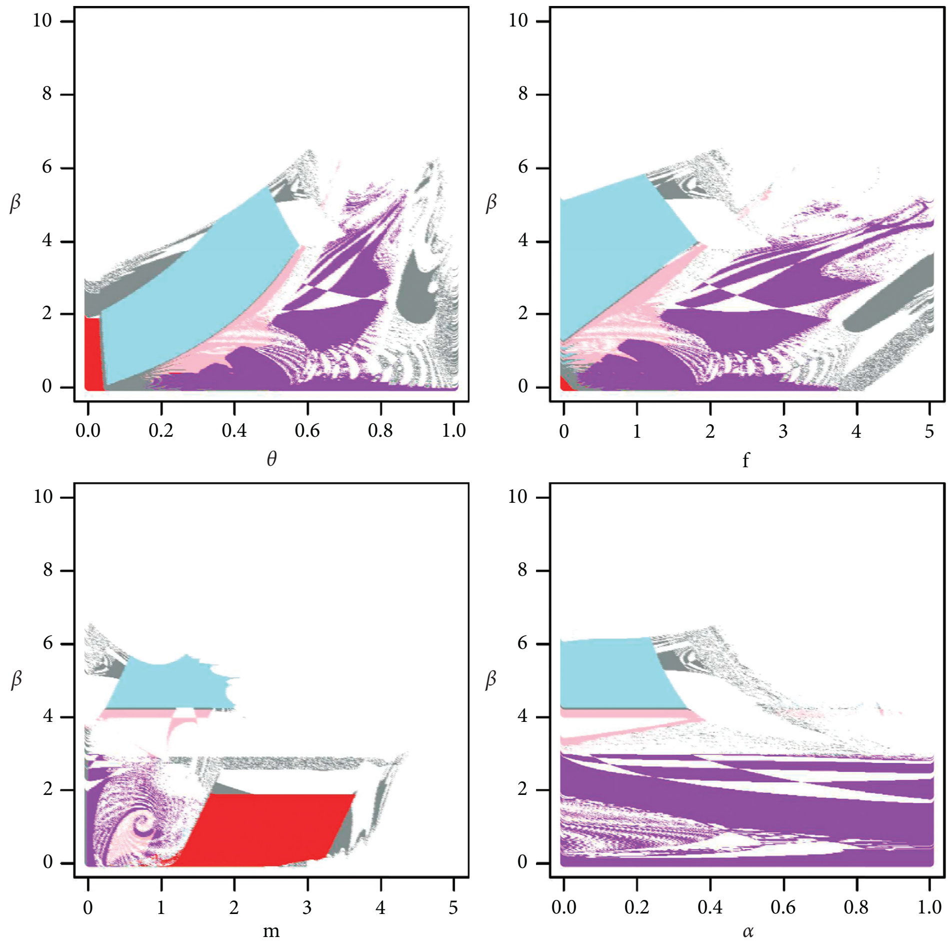

Figure 14: 2D bifurcation diagram. $w = 1$, $w_0 = 0.1$. When they are not the parameter under variation, their values are $\theta = 0.6$, $f = 2$, $m = 0.1$, $\alpha = 0.4$.

β and the proportion of the recovered earnings α. When β is high, then the path is towards corrupt agents and a monitoring firm.

6. Concluding Remarks and Future Research

The main aim of this work was to find out under which conditions what shares of honest agents and what probabilities of monitoring are evolutionary stable. Hence, we first constructed a static model with an associated asymmetric payoff-matrix which allowed us to infer hypotheses about the growth of these groups. Second, we use the same payoff-matrix to build an evolutionary model, using the replicator dynamics model. Formal analyses of the model revealed the existence of five possible equilibrium points, which do not however always exist and which are not always evolutionary stable.

The complexity behavior of the system is studied by using 1D and 2D bifurcation diagrams, chaotic attractors, and limit cycle. Finally, we make effective control of the chaotic behaviors in the system by using control variables.

From this analysis we can put forward the following conclusions:

(1) Despite the fact that corrupt agents and a nonmonitoring firm are stable, in practice this happens only when the probability of detecting wrongdoing is low and the rate at which the system converges to the equilibrium point is slow, taking infinite time to reach the equilibrium, showing an organization that is always in a transient state. In this case, it is not possible to predict the proportion of honest agents the organization will have and since the probability of detecting wrongdoing is small, the firm will not monitor at all.

(2) An equilibrium of corrupt agents and a monitoring firm is stable whenever the earnings per wrongdoing are high and the penalty is in the low to medium range. When the penalty is high enough, the equilibrium of corrupt agents and a monitoring firm becomes no longer stable, moving towards a neutrally stable system, where the center is given by a proportion of honest agents p^* and the probability of monitor is s^*. When the probability of detecting wrongdoing is small and the conditions of stability for the equilibrium of corrupt agents and a monitoring firm are satisfied, again it may happen that the rate at which the system converges to the equilibrium point is slow, taking infinite time to reach the equilibrium, showing an organization that is always

in a transient state, where the proportion of honest agents behaves chaotically, while the firm monitors.

(3) A culture of honest agents, with or without monitoring, is never evolutionary stable. Nevertheless, when the penalty is high enough, the system is neutrally stable to $E_3 = (1 - (m/\theta(\alpha\beta + f)), (\beta/\theta(w + \beta + f - w_0)))$, which converges to $E = (1, 0)$ when $f \longrightarrow \infty$.

It is not surprising that an equilibrium of honest agents (E_4 and E_5) is not stable, since in an asymmetric game only pure-strategy Nash equilibria can be asymptotically stable. Nash equilibrium is a situation in which an individual has no incentives to change his/her strategies, assuming the other players remain constant in their actions. Thus, the results found point to the fact that there is not a clear cut solution to defeat crime. This is not surprising, as crime has been hard-to-break behavior since the beginning of time.

There is further analysis that could be undertaken in this field. We would like to highlight two future avenues of research. Firstly, it is relevant to differentiate between petty crime (that possibly many people do) and serious crimes, which are more important from a deterrence point of view. These are different types of crimes that require different deterrence strategies, which obviously in turn imply different dynamics. Secondly, it could be of great interest, to use the present dynamic setting to model the case in which individuals can commit more than one offense (e.g., up to two). In the static literature, a different fine has been suggested for the second offense. Indeed, it is argued that, in some cases, it might be better to punish repeat offenders more severely than first-time offenders, while in another cases, it might be better to impose less severe penalties on repeat offenders; see [33]. To explore these assumptions in a dynamic setting could therefore contribute to the development of the subject.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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